# 1 Number System Basics

Digital technology has become widespread and encompasses virtually all aspects of our everyday lives. We could see it being used in computers and related gadgets, entertainment, automation (robotics), medical etc. Though physical quantities measured in the real world are analogue, most of these are processed by digital means. In order to do this, we have to convert the measured analogue quantity into digital, process the digital quantity using digital circuitry and then reconvert to analogue.

The contents of this book concentrate on the digital circuit design to enable the processing of the digital quantity. But before we look into the principles of such designs, we need to understand the basics of number systems.

# 1.1 Decimal Numbers

Decimal number system is the commonly used number system that has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. It is also known as base (or radix) ten system since it has ten digits that can be used to represent any number. Figure 1.1 shows the positional values or weights of the decimal number system for an integer.



Figure 1.1: Decimal number system for integers.

The digit with least weight (i.e. the one on the foremost right) is known as the least significant digit (LSD) while the highest weight digit is known as the most significant digit (MSD). In the example shown in Figure 1.1, the MSD is digit 6 while the LSD is digit 3. Figure 1.2 shows the case for fractional decimal number.



Figure 1.2: Decimal number system for fractional numbers.

## 1.2 Other Number Systems – Binary, Octal and Hexadecimal

While decimal number system is the commonly used number system in everyday lives, digital devices uses only binary number system that consists of 0 and 1. The base is two for this system and Figure 1.3 show an example of binary number for decimal equivalent of  $6.25_{10}$ 





Figure 1.3: Binary number system with an example.

Similarly, octal and hexadecimal (hex in short) number systems have number bases of 8 and 16. For octal number system, the eight digits are 0, 1, 2, 3, 4, 5, 6, and 7 while hexadecimal number system has 16 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F. Figure 1.4 gives examples on these number systems.



(a)



(b)

Figure 1.4: Number system examples (a) octal (b) hex.



## 1.3 Conversion between different number systems

It is often necessary to convert a number from one base system to another. Converting a number to decimal is rather straightforward as we have seen in the previous examples. The weights or positional values (for the appropriate base) are multiplied with the digit and summed to give the decimal value. In this section, we will look at methods to convert numbers from decimal to binary, octal and hex. Other conversions such as octal to binary (and vice versa), binary to hex, hex to binary, octal to hex and hex to octal are also possible.

#### 1.3.1 Decimal to binary, octal and hex conversions

There are two methods that can be used to achieve decimal to binary conversion. The first method is by presenting the decimal value in units, tens, hundreds etc. For example:

$$26 = 16 + 8 + 2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 11010_{10}$$

The problem with this method is that certain positional values (such as  $2^2$  and  $2^0$  in the example above) can easily be forgotten. There is another method called repeated division that is more frequently employed. Figure 1.5 illustrates this method. It works by repeated division with a value of 2 (until the quotient is 0) and the remainder digits from each step represent the binary number (in reverse order).



**Figure 1.5:** Decimal to binary conversion example,  $34_{10} = 100010_2$ .

Similarly, we can convert a decimal number to octal and hex. Figures 1.6 and 1.7 illustrate the steps for these conversions. Do remember that the final answer is in the reverse order!



**Figure 1.6:** Decimal to octal conversion example,  $149_{10} = 225_8$ .



**Figure 1.7:** Decimal to hex conversion example,  $564_{10} = 234_{16}$ .

#### 1.3.2 Binary to Octal and vice versa

Any binary number can be converted to octal simply by grouping them in groups of three digits. For example,  $100101110_8$  can be converted to  $456_8$  as shown in Figure 1.8 (a). The reverse procedure of converting an octal number to binary can be done by writing three binary digit equivalent for each octal digit. This is shown in Figure 1.8 (b).



**Figure 1.8**: Octal to binary conversion example and vice versa: (a)  $100101110_2 = 456_8$  (b)  $752_8 = 111101010_2$ .

#### 1.3.3 Binary to Hex and vice versa

Similar to octal number, binary number can be converted to hex simply by grouping them in groups of four digits. For example,  $10010111_2$  can be converted to  $97_{16}$  as shown in Figure 1.9 (a). A hex number can be converted to binary by writing four binary digit equivalent for each hex digit. This is shown in Figure 1.9 (b).



**Figure 1.9:** Hex to binary conversion example and vice versa: (a)  $10010111_2 = 97_{16}$  (b)  $832_{16} = 100000110010_2$ .

## 1.4 Other number codes

In this section, several other commonly used codes will be discussed.

#### 1.4.1 ASCII code

ASCII stands for American Standard Code for Information Interchange. Characters such as 'a', 'A', '@', '\$' each have a code that is recognised by the computer. Standard ASCII has 128 characters (represented by 7 binary digits;  $2^7$ =128), though the first 32 is no longer used. Extended ASCII has another 128 characters, mostly to represent special characters and mathematical symbols such as ' $\ddot{y}$ , ' $\ddot{e}$ , ' $\Sigma$ ', and ' $\sigma$ '. Table 1.1 shows the standard ASCII code.

D/mal	Нох	P'any	Char	D/mal	Цах	P'any	Char	D/mal	Цах	P'any	Char
D Mai	нех	Bary	Char	D mai	Hex	Вагу	Char	D mai	Hex	Bary	Char
32	20	0100000	space	48	30	0110000	0	64	40	1000000	@
33	21	0100001	!	49	31	0110001	1	65	41	1000001	A
34	22	0100010	"	50	32	0110010	2	66	42	1000010	В
35	23	0100011	#	51	33	0110011	3	67	43	1000011	с
36	24	0100100	\$	52	34	0110100	4	68	44	1000100	D
37	25	0100101	%	53	35	0110101	5	69	45	1000101	E
38	26	0100110	&	54	36	0110110	6	70	46	1000110	F
39	27	0100111	1	55	37	0110111	7	71	47	1000111	G
40	28	0101000	(	56	38	0111000	8	72	48	1001000	н
41	29	0101001	)	57	39	0111001	9	73	49	1001001	1
42	2A	0101010	*	58	3A	0111010	:	74	4A	1001010	J
43	2B	0101011	+	59	ЗB	0111011	;	75	4B	1001011	к
44	2C	0101100	,	60	3C	0111100	<	76	4C	1001100	L
45	2D	0101101	-	61	3D	0111101	=	77	4D	1001101	М
46	2E	0101110		62	3E	0111110	>	78	4E	1001110	N
47	2F	0101111	/	63	3F	0111111	?	79	4F	1001111	0
							Į.	<u>.</u>			
D'mal	Hex	B'ary	Char	D'mal	Hex	B'ary	Char	D'mal	Hex	B'ary	Char
D'mal	Hex 50	B'ary 1010000	Char P	D'mal 96	Hex 60	B'ary 1100000	Char	D'mal	Hex 70	B'ary 1110000	Char p
D'mal 80 81	Hex 50 51	B'ary 1010000 1010001	Char P Q	D'mal 96 97	Hex 60 61	B'ary 1100000 1100001	Char ` a	D'mal 112 113	Hex 70 71	B'ary 1110000 1110001	Char p q
D'mal 80 81 82	Hex 50 51 52	B'ary 1010000 1010001 1010010	Char P Q R	D'mal 96 97 98	Hex 60 61 62	B'ary 1100000 1100001 1100010	Char ` a b	D'mal 112 113 114	Hex 70 71 72	B'ary 1110000 1110001 1110010	Char p q r
D'mal 80 81 82 83	Hex 50 51 52 53	B'ary 1010000 1010001 1010010 1010011	Char P Q R S	D'mal 96 97 98 99	Hex 60 61 62 63	B'ary 1100000 1100001 1100010 1100011	Char ` a b c	D'mal 112 113 114 115	Hex 70 71 72 73	B'ary 1110000 1110001 1110010 1110011	Char p q r s
D'mal 80 81 82 83 84	Hex 50 51 52 53 54	B'ary 1010000 1010001 1010010 1010011 1010100	Char P Q R S T	D'mal 96 97 98 99 100	Hex 60 61 62 63 64	B'ary 1100000 1100001 1100010 1100011 1100100	Char ` a b c d	D'mal 112 113 114 115 116	Hex 70 71 72 73 74	B'ary 1110000 1110001 1110010 1110011 1110100	Char p q r s t
D'mal 80 81 82 83 84 85	Hex 50 51 52 53 54 55	B'ary 1010000 1010001 1010010 1010011 1010100 1010101	Char P Q R S T U	D'mal 96 97 98 99 100 101	Hex 60 61 62 63 64 65	B'ary 1100000 1100001 1100010 1100011 1100100	Char ` a b c d e	D'mal 112 113 114 115 116 117	Hex 70 71 72 73 74 75	B'ary 1110000 1110001 1110010 1110011 1110100 11110101	Char p q r s t u
D'mal 80 81 82 83 84 85 86	Hex 50 51 52 53 54 55 55 56	B'ary 1010000 1010001 1010010 1010011 1010100 1010101	Char P Q R S T U V	D'mal 96 97 98 99 100 101 102	Hex 60 61 62 63 64 65 66	B'ary 1100000 1100001 1100010 1100011 1100100	Char a b c d e f	D'mal 112 113 114 115 116 117 118	Hex 70 71 72 73 74 75 76	B'ary 1110000 1110001 1110010 1110011 1110100 1110101 1110110	Char p q r s t u v
D'mal 80 81 82 83 84 85 86 86 87	Hex 50 51 52 53 54 55 56 56 57	B'ary 1010000 1010001 1010010 1010011 1010100 1010110 1010111	Char P Q R S T U V V W	D'mal 96 97 98 99 100 101 102 103	Hex 60 61 62 63 64 65 66 66 67	B'ary 1100000 1100001 1100010 1100011 1100100	Char a b c d e f g	D'mal 112 113 114 115 116 117 118 119	Hex 70 71 72 73 74 75 76 77	B'ary 1110000 1110001 1110010 1110011 1110100 1110110	Char p q r s t u v v w
D'mal 80 81 82 83 84 85 86 86 87 88	Hex 50 51 52 53 54 55 56 57 58	B'ary 1010000 1010001 1010010 1010011 1010100 1010110 1010111 1011000	Char P Q R S T U U V W X	D'mal 96 97 98 99 100 101 102 103 104	Hex 60 61 62 63 64 65 66 66 67 68	B'ary 1100000 1100001 1100010 1100011 1100100	Char a b c d e f g h	D'mal 112 113 114 115 116 117 118 119 120	Hex 70 71 72 73 74 75 76 76 77 78	B'ary 1110000 1110001 1110010 1110011 1110100 1110110	Char p q r s t u v v w x
D'mal 80 81 82 83 84 85 86 85 86 87 88 88 89	Hex 50 51 52 53 54 55 56 57 58 58 59	B'ary 1010000 1010001 1010010 1010011 1010100 1010110 1010111 1011000 1011001	Char P Q R S T U V V W X X Y	D'mal 96 97 98 99 100 101 102 103 104 105	Hex 60 61 62 63 64 65 66 67 68 68 69	B'ary 1100000 1100001 1100010 1100011 1100100	Char a b c d e f g h i	D'mal 112 113 114 115 116 117 118 119 120 121	Hex 70 71 72 73 74 75 76 77 78 78 79	B'ary 1110000 1110001 1110010 1110011 1110100 1110110	Char p q r s t u v v w x y
D'mal 80 81 82 83 84 85 86 85 86 87 88 88 89 90	Hex 50 51 52 53 54 55 56 57 58 59 59 5A	B'ary 1010000 1010001 1010010 1010011 1010100 1010111 1011000 1011001 1011001	Char P Q R S T U U V W X X Y Z	D'mal 96 97 98 99 100 101 102 103 104 105 106	Hex 60 61 62 63 64 65 66 67 68 69 6A	B'ary 1100000 1100001 1100010 1100011 1100100	Char a b c d e f g h i j	D'mal 112 113 114 115 116 117 118 119 120 121 122	Hex 70 71 72 73 74 75 76 77 78 79 78	B'ary 1110000 1110001 1110010 1110011 1110100 1110110	Char p q r s t u v w x y z
D'mal 80 81 82 83 84 85 86 87 88 88 89 90 91	Hex 50 51 52 53 54 55 56 57 58 59 58 59 5A 5B	B'ary 1010000 1010001 1010010 1010011 1010100 1010111 1011000 1011001 1011001 101101	Char P Q R S T U V V W X Y Z [	D'mal 96 97 98 99 100 101 102 103 104 105 106 107	Hex 60 61 62 63 64 65 66 67 68 69 68 69 6A 6B	B'ary 1100000 1100001 1100010 1100011 1100100	Char a b c d e f g h i j k	D'mal 112 113 114 115 116 117 118 119 120 121 122 123	Hex 70 71 72 73 74 75 76 77 78 78 79 7A 78	B'ary 1110000 1110001 1110010 1110011 1110100 1110101 1110110	Char p q r s t u v w x y z {
D'mal 80 81 82 83 84 85 86 87 88 88 89 90 91 92	Hex 50 51 52 53 54 55 56 57 58 57 58 59 58 59 5A 58 58 59 5A 58 55	B'ary 1010000 1010001 1010010 1010011 1010100 1010110 1011000 1011001 1011001 101101	Char P Q R S T U V W X Y Z [ \	D'mal 96 97 98 99 100 101 102 103 104 105 106 107 108	Hex 60 61 62 63 64 65 66 67 68 69 68 69 6A 6B 6B 6C	B'ary 1100000 1100001 1100010 1100011 1100100	Char a b c d e f g h i j k l	D'mal 112 113 114 115 116 117 118 119 120 121 122 123 124	Hex 70 71 72 73 74 75 76 77 78 78 79 78 79 7A 78 79 7A	B'ary 1110000 1110001 1110010 1110011 1110100 1110101 1110100 1111001 1111001 1111001 1111010	Char p q r s t u v w x y z { 
D'mal 80 81 82 83 84 85 86 87 88 88 89 90 91 92 93	Hex 50 51 52 53 54 55 56 57 58 57 58 59 58 59 58 59 5A 58 59 50 5D	B'ary 1010000 1010001 1010010 1010011 1010100 1010101 1010110 1011000 1011001 101101	Char P Q R S T U V W X Y Z [ \ ]	D'mal 96 97 98 99 100 101 102 103 104 105 106 107 108 109	Hex 60 61 62 63 64 65 66 67 68 69 68 69 6A 6B 6B 6C 6D	B'ary 1100000 1100011 1100010 1100101 1100100 1100101 1100101 1101000 1101001 1101010 1101101 1101100 1101101	Char a b c d d e f g h i j k l m	D'mal 112 113 114 115 116 117 118 119 120 121 122 123 124 125	Hex 70 71 72 73 74 75 76 77 76 77 78 79 78 79 7A 78 79 7A 7B 7C	B'ary 1110000 1110001 1110010 1110011 1110100 1110101 1110100 1111000 1111001 1111010 1111000 1111101	Char p q r s t u v w x y z { l }
D'mal 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94	Hex 50 51 52 53 54 55 56 57 58 57 58 59 58 59 5A 58 59 5A 5B 5C 5D 5E	B'ary 1010000 1010001 1010010 1010011 1010100 1010101 1010110 1011000 1011001 1011001 1011000 1011101 1011100	Char P Q R S T U V W X Y Z [ \ 1 ^	D'mal 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110	Hex 60 61 62 63 64 65 66 67 68 67 68 69 68 69 6A 6B 6C 6D 6E	B'ary 1100000 1100001 1100010 1100101 1100100 1100101 1100101 1101000 1101001 1101001 1101100 1101101 1101101 1101110	Char a b c d e f g h i j k l m n	D'mal 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126	Hex 70 71 72 73 74 75 76 75 76 77 78 79 78 79 78 79 7A 79 7A 79 7A 79 7A 70 72	B'ary 1110000 1110001 1110010 1110011 1110100 1110101 1110100 1111001 1111000 1111001 1111000 1111101 1111100	Char p q r s t u v w x y z { l } ~

#### Table 1.1: Standard ASCII code

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## 1.4.2 Binary coded decimal (BCD)

BCD is actually a set of binary numbers where a group of four binary numbers represent a decimal digit. As there are 10 basic digits in the decimal number system, four binary digits (bits) are required<sup>1</sup>. Figure 1.10 shows an example, while Table 1.2 gives the BCD code.



Figure 1.9: Hex to binary conversion example and vice versa: 973<sub>10</sub> = 10010111.0011<sub>BCD.</sub>

1 Three bits will only give eight representations, which is not enough for a decimal system.



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#### Table 1.2: BCD code

Decimal	BCD	Decimal	BCD
0	0000	5	0101
1	0001	6	0110
2	0010	7	0111
3	0011	8	1000
4	0100	9	1001

#### 1.4.3 Gray code

Gray code is another commonly encountered code system. The main feature of this code is that only one bit changes between two successive values. This system is less prone to errors and is considered very useful for practical applications such as mechanical switches and error correction in digital communication as compared to the standard binary system. Table 1.3 gives the BCD code with 4 bits (i.e. up to decimal value of 15).

Decimal	Gray	Decimal	Gray
0	0000	8	1100
1	0001	9	1101
2	0011	10	1111
3	0010	11	1110
4	0110	12	1010
5	0111	13	1011
6	0101	14	1001
7	0100	15	1000

#### Table 1.3: Gray code